

Assimilation of GPS delays with COSMO-DE/Kenda

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Deutscher Wetterdienst
Wetter und Klima aus einer Hand



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COSMO-DE

Resolution: 2.8 km

Operational assimilation: nudging

Boundary conditions: ICON EU nest

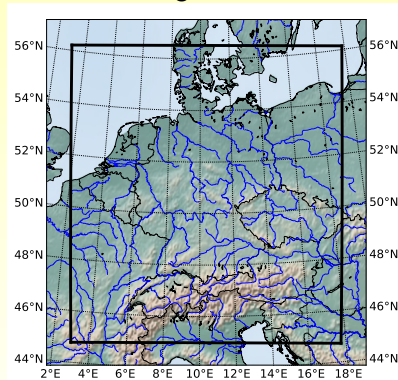
LETKF

The local transform ensemble Kalman Filter (LETKF) is in a pre-operational state and can be used for assimilation experiments.

STD-Operator

The STD/ZTD operator is called within each COSMO time step to compute the model equivalents of the observations.

COSMO-DE region



1h COSMO-DE forecast, $\Delta t = 25$ s, observation operator \mathcal{H} :

\mathbf{x}_a, \mathbf{y}	$\mathbf{x}(t_1)$	$\mathbf{x}(t_2)$	\xrightarrow{t}	$\mathbf{x}(t_{n-1})$	$\mathbf{x}(t_n)$	\mathbf{x}_f
	\Downarrow	\Downarrow	\dots	\Downarrow	\Downarrow	
	$\mathcal{H}(\mathbf{x}(t_1))$	$\mathcal{H}(\mathbf{x}(t_2))$	\dots	$\mathcal{H}(\mathbf{x}(t_{n-1}))$	$\mathcal{H}(\mathbf{x}(t_n))$	$\Rightarrow \mathcal{H}(\mathbf{x})$

Ensemble of 40 COSMO-DE 1 h forecasts:

member 1 $\mathbf{x}_f^1, \mathbf{y}, \mathcal{H}(\mathbf{x}^1)$	member 2 $\mathbf{x}_f^2, \mathbf{y}, \mathcal{H}(\mathbf{x}^2)$	\dots	member 39 $\mathbf{x}_f^{39}, \mathbf{y}, \mathcal{H}(\mathbf{x}^{39})$	member 40 $\mathbf{x}_f^{40}, \mathbf{y}, \mathcal{H}(\mathbf{x}^{40})$
\Downarrow				
$\text{LETKF} (\mathbf{x}_f^i, \mathbf{y} - \mathcal{H}(\mathbf{x}^i))$				
\Downarrow				
\mathbf{x}_a^1	\mathbf{x}_a^2	\dots	\mathbf{x}_a^{39}	\mathbf{x}_a^{40}



forecasts

The STD assimilation operator H_{STD} consists of 2 parts which are called by the ICON En-Var or COSMO interface:

1) Setup of the observation geometry

- Computes connecting line between satellite and receiver
- Definition of supporting points on the signal path

2) Signal path estimation and delay computation

- Interpolation/extrapolation of the model data on the supporting points
- Call of the raytracer, iterative estimation of the signal path
- Delay computation - integration of the refractive index along the signal path

The interface handles data I/O, load balancing, MPI data exchange, ...

Experiment Summer 2014

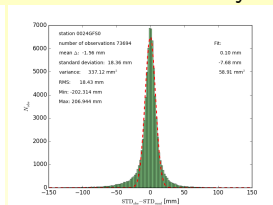
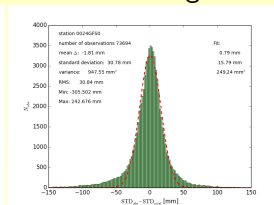
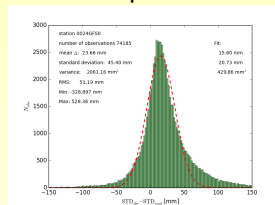
Period	16.5.2014 – 1.6.2014
NWP system	COSMO-DE Kenda-LETKF
Initial conditions	ICON EU nest, 16.5.2014, 0:00 UTC
Boundary conditions	ICON EU nest, every 3 hours
Control experiment	assimilation of conventional observations, GPS STDs passive
GPS experiment	assimilation of conventional observations and GPS STD observations
Observation error	10 mm mapped on slant path
GPS data	STD data provided by GFZ, Potsdam
GPS stations	~ 300 within the COSMO-DE region
GPS observations	~ 50000 per hour, $20 \cdot 10^6$ total

Station 0024 (SAPOS), Eberswalde, Germany, $h = 24$ m

control experiment

obs. - first guess

observation - analysis



$$\begin{aligned} N_{obs} &= 74185 \\ \text{bias} &= 23.7 \text{ mm} \\ \sigma &= 45.4 \text{ mm} \end{aligned}$$

$$\begin{aligned} N_{obs} &= 73694 \\ \text{bias} &= -1.8 \text{ mm} \\ \sigma &= 30.8 \text{ mm} \end{aligned}$$

$$\begin{aligned} N_{obs} &= 73694 \\ \text{bias} &= -1.6 \text{ mm} \\ \sigma &= 18.4 \text{ mm} \end{aligned}$$

$$\begin{aligned} N_{obs} &= 20178949 \\ \sigma &= 41.6 \text{ mm} \\ \sigma_{map} &= 13.6 \text{ mm} \end{aligned}$$

$$\begin{aligned} N_{obs} &= 19966361 \\ \sigma &= 27.9 \text{ mm} \\ \sigma_{map} &= 9.2 \text{ mm} \end{aligned}$$

$$\begin{aligned} N_{obs} &= 19966361 \\ \sigma &= 17.3 \text{ mm} \\ \sigma_{map} &= 5.4 \text{ mm} \end{aligned}$$

⇒ almost all STD data assimilated (~ 99 %)

28.5.2014, 1:00 UTC, 0:00 UTC forecast, 1 mm/h threshold

radar observations

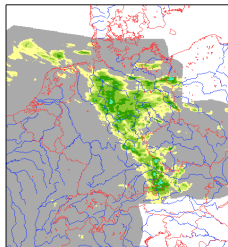
control experiment

STD assimilation

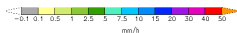
RADAR COMPOSITE

valid: 28 MAY 2014 00 - 01 UTC

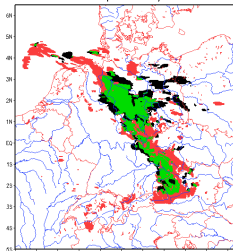
1h PRECIPITATION



Mean: 0.240524 Min: 0 Max: 9.59667

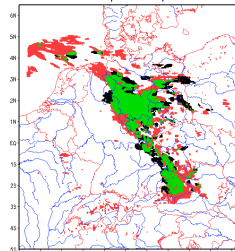


exp_2000.01_MBn_2014052800+01h
Precip>1.0 mm/h



Radar: mean: 0.191 mm/h max: 9.596 mm/h
Model: mean: 0.231 mm/h max: 23.99 mm/h
missed (black): 5217
false (red): 9299 hit (green): 6511
ETS: 0.263 FBI: 1.340

exp_2000.03_MBn_2014052800+01h
Precip>1.0 mm/h



Radar: mean: 0.191 mm/h max: 9.596 mm/h
Model: mean: 0.275 mm/h max: 24.00 mm/h
missed (black): 4088
false (red): 9861 hit (green): 7640
ETS: 0.307 FBI: 1.492

	hit	miss	false	ETS
control experiment	6511	5217	9299	0.283
STD assimilation	7640	4088	9861	0.307

12 UTC forecast, precipitation threshold 1.0 mm/h

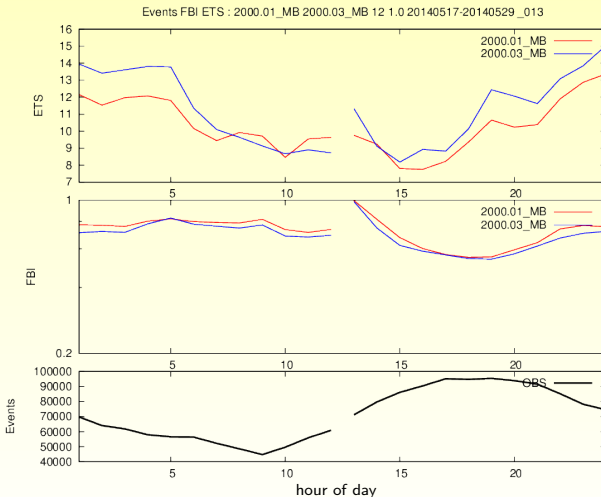
equitable threat score

$$ETS = \frac{A - A_r}{A + B + C - A_r}$$

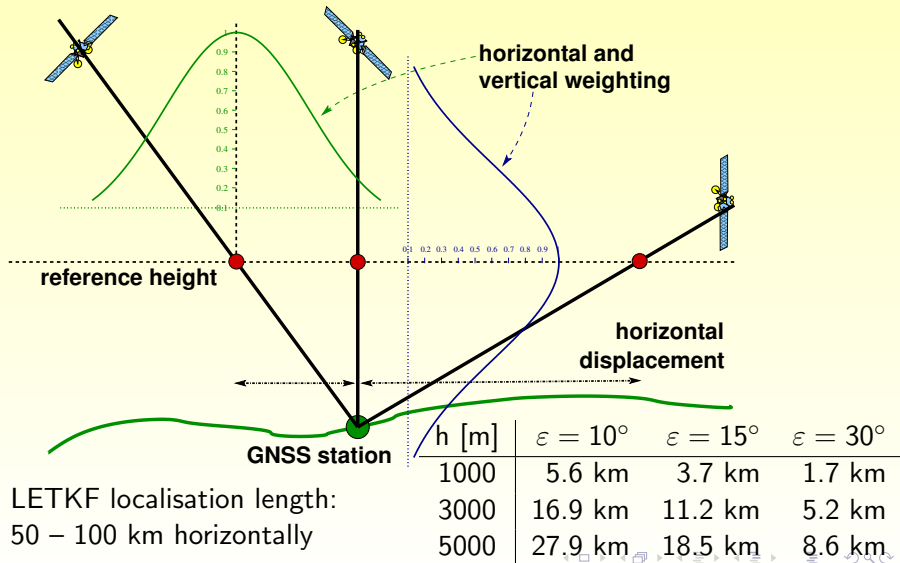
frequency bias

$$FBI = \frac{A+B}{A+C}$$

event - COSMO grid
node with rain above
threshold



Local Transform Ensemble Kalman Filter → localisation of STDs?



Station Marburg – STD at station coordinates

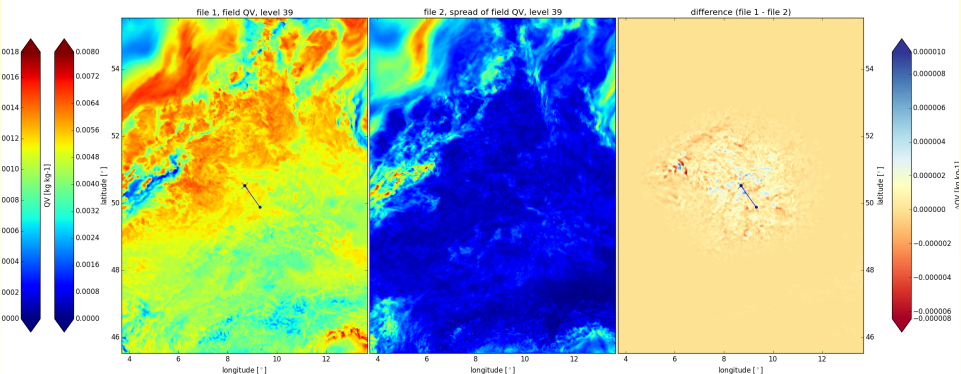
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

QV field

spread

increment



COSMO-DE QV field, 10 m layer

Station Marburg – STD at station coordinates

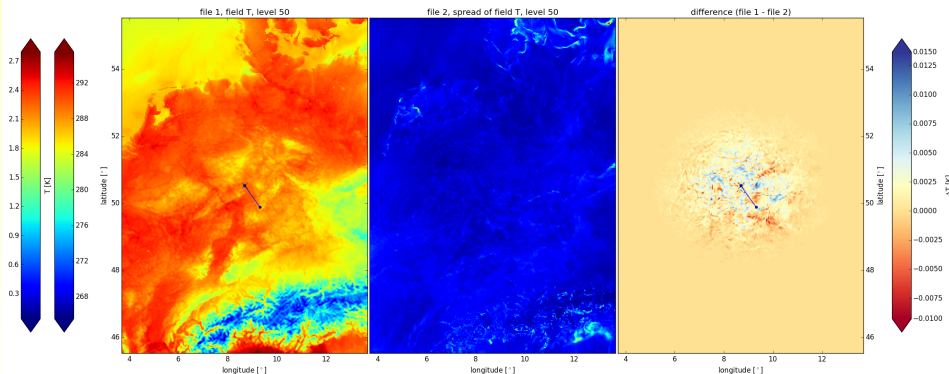
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

T field

spread

increment



COSMO-DE T field, 10 m layer

Station Marburg – STD at station coordinates

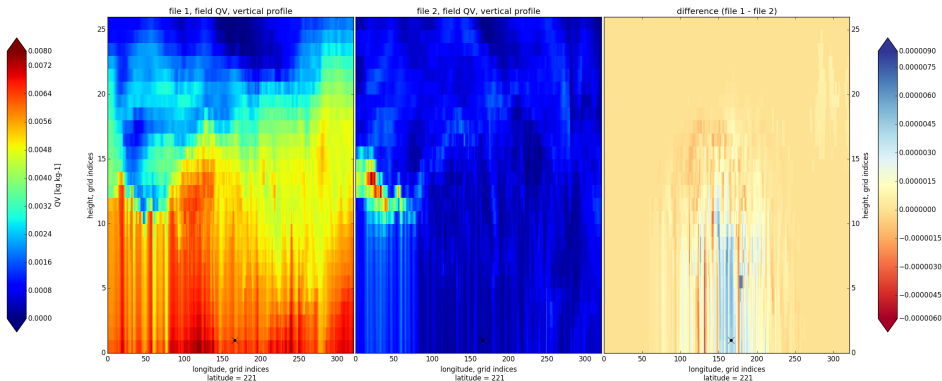
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

QV field

spread

increment



COSMO-DE QV field, vertical slice up to 5000 m (index 25), $i12 = 990$ m

Station Marburg – STD at station coordinates

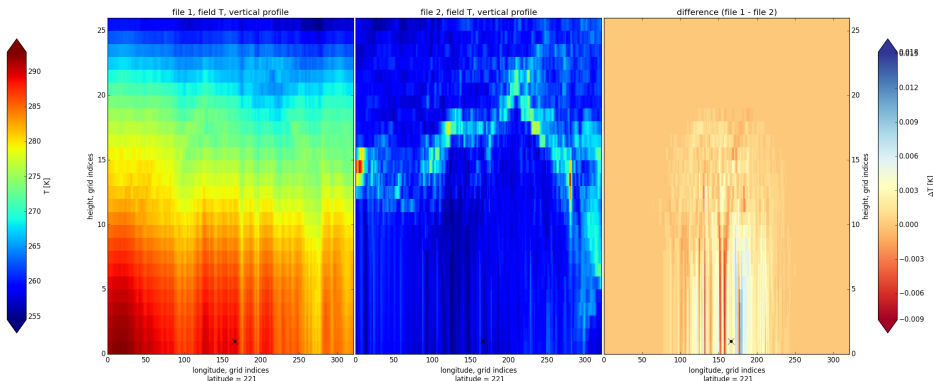
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

T field

spread

increment



COSMO-DE T field, vertical slice up to 5000 m (index 25), $i12 = 990$ m

Station Marburg – STD 2000 m above station

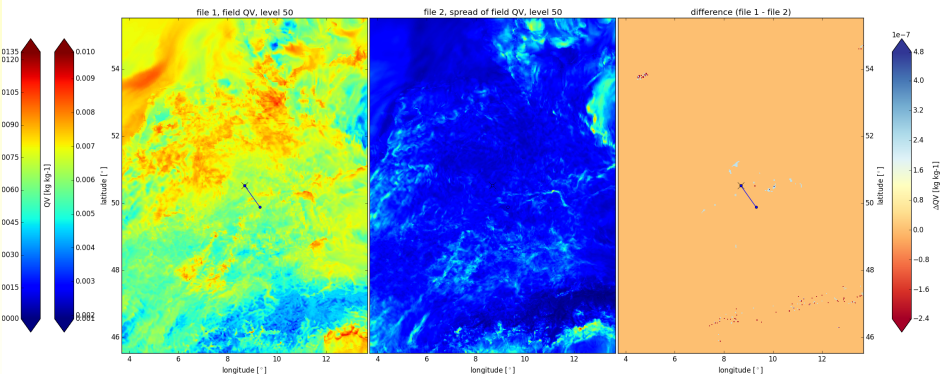
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

QV field

spread

increment



COSMO-DE QV field, 10 m layer

Station Marburg – STD 2000 m above station

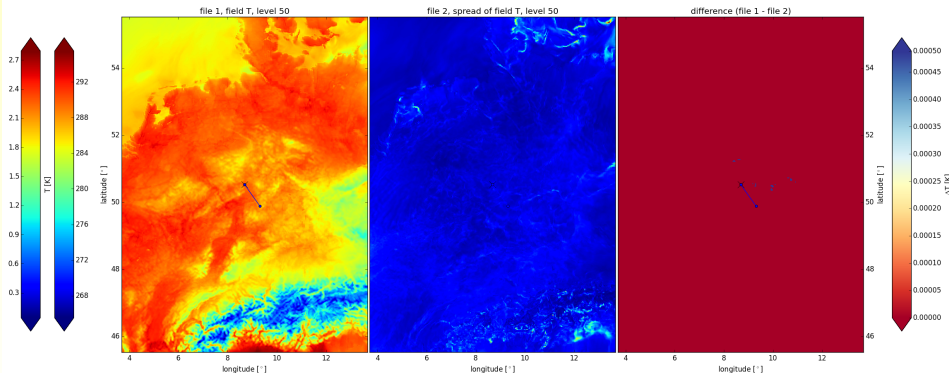
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

T field

spread

increment



COSMO-DE T field, 10 m layer

Station Marburg – STD 2000 m above station

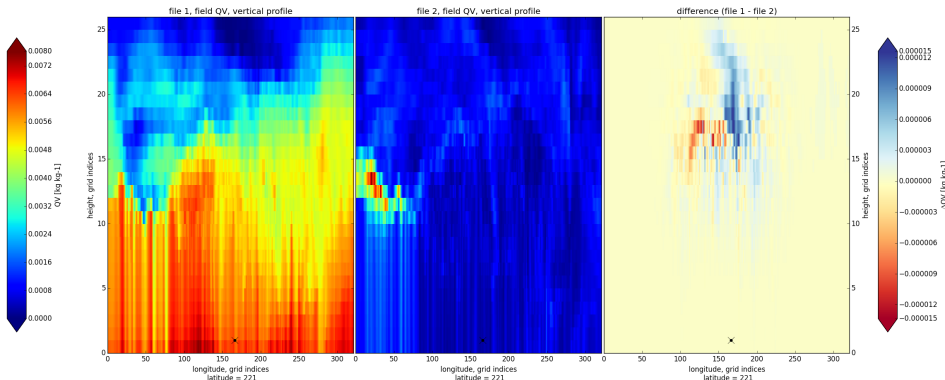
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

QV field

spread

increment



COSMO-DE QV field, vertical slice up to 5000 m (index 25), $i17 = 2014$ m

Station Marburg – STD 2000 m above station

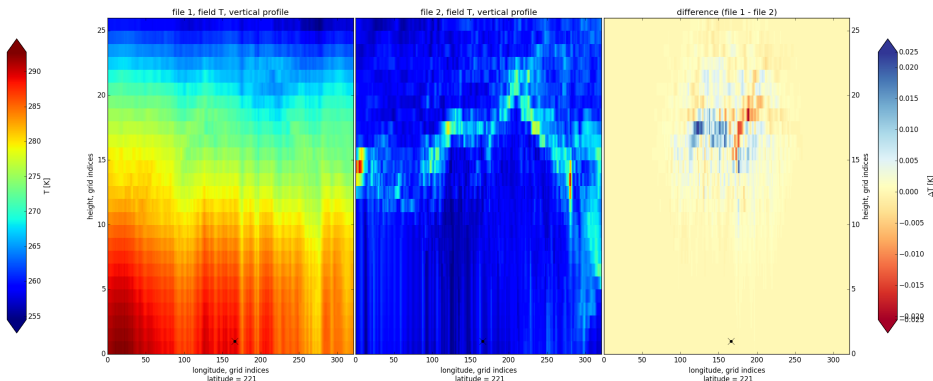
Latitude: $8,685^\circ$, longitude: $50,533^\circ$, height ASL: 222 m

STD = 9.5403 m, azimuth = 148.4° , elevation = 14.15°

T field

spread

increment



COSMO-DE T field, vertical slice up to 5000 m (index 25), $i17 = 2014$ m

Analysis \mathbf{x}_a :

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}(\mathbf{y}_0 - \mathcal{H}(\mathbf{x}_b))$$

with the Kalman-Gain-Matrix \mathbf{K}

$$\begin{aligned}\mathbf{K} &= (\mathbf{B}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{R}^{-1} \\ &= \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1}\end{aligned}$$

Analysis increments $\delta \mathbf{x}_a$

$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \delta \mathbf{y}_0$$

Increments $\delta \mathbf{x}_a$ are large, where \mathbf{H} is large:

$$\mathbf{H} = (h_{ij}) \quad , \quad h_{ij} = \frac{\partial \mathcal{H}_i}{\partial x_j}$$

$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \delta \mathbf{y}_0$$

For one single STD observation $\delta \mathbf{x}_a$ can be computed:

- $\delta \mathbf{y}_0 = \mathbf{y}_0 - \mathcal{H}(\mathbf{x}_b) = STD - \mathcal{H}(\mathbf{x}_b) \rightarrow$ real number
- $\mathbf{R} = \varepsilon_{STD} \rightarrow$ real number
- $\mathbf{H} = \left(\frac{\partial STD}{\partial \mathbf{x}_j} \right) \rightarrow$ vector, provided by adjoint operator
- $\mathbf{B} \rightarrow$ matrix, provided by ensemble

\mathbf{B} is still a large matrix as p, T and qv on all grid nodes in the vicinity of the slant path need to be regarded \rightarrow some GB.

B can be estimated from an ensemble $\mathbf{x}^{b(k)}$ of state vectors

$$\mathbf{B} = \frac{1}{K-1} \sum_{k=1}^K \left(\mathbf{x}^{b(k)} - \bar{\mathbf{x}}^b \right) \left(\mathbf{x}^{b(k)} - \bar{\mathbf{x}}^b \right)^T$$

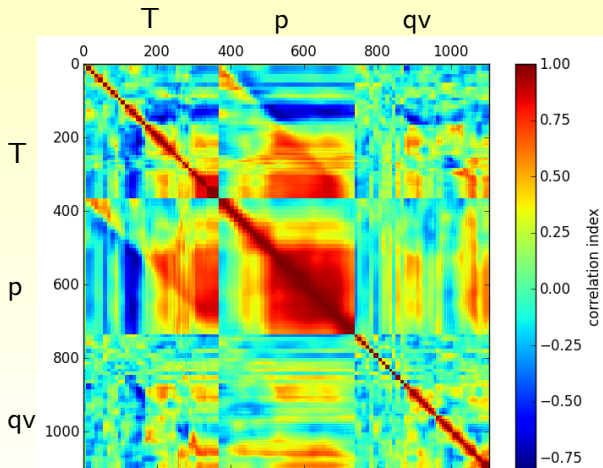
The matrix elements b_{ij} of **B** are given by

$$b_{ij} = \frac{1}{K-1} \sum_{k=1}^K \left(x_i^{b(k)} - \bar{x}_i^b \right) \left(x_j^{b(k)} - \bar{x}_j^b \right)$$

The correlation matrix **D** is much more descriptive and can be computed from **B**:

$$d_{ij} = \frac{b_{ij}}{\sqrt{b_{ii} \cdot b_{jj}}}$$

STD: 10.39 m, elevation = 13.35° , azimuth = 132.30°



\mathcal{H} acts on
368 grid nodes



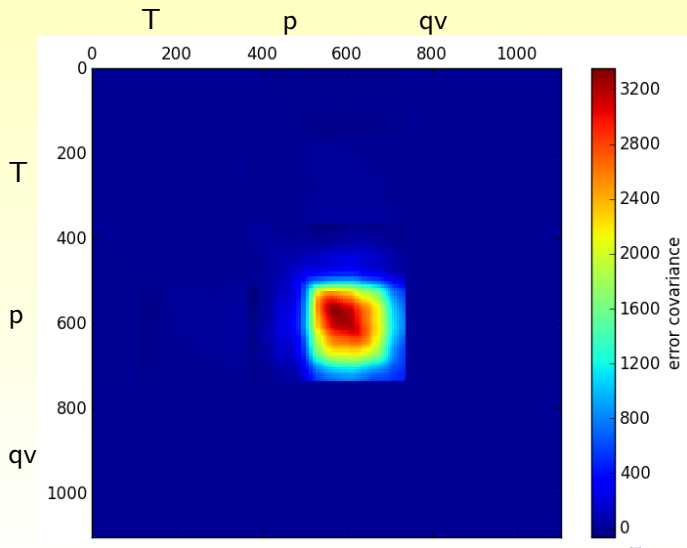
3 values per node:
p, T, qv



dimension of
state vector \mathbf{x} :
 $368 \cdot 3 = 1104$

Background Error Covariance Matrix

STD: 10.39 m, elevation = 13.35°, azimuth = 132.30°



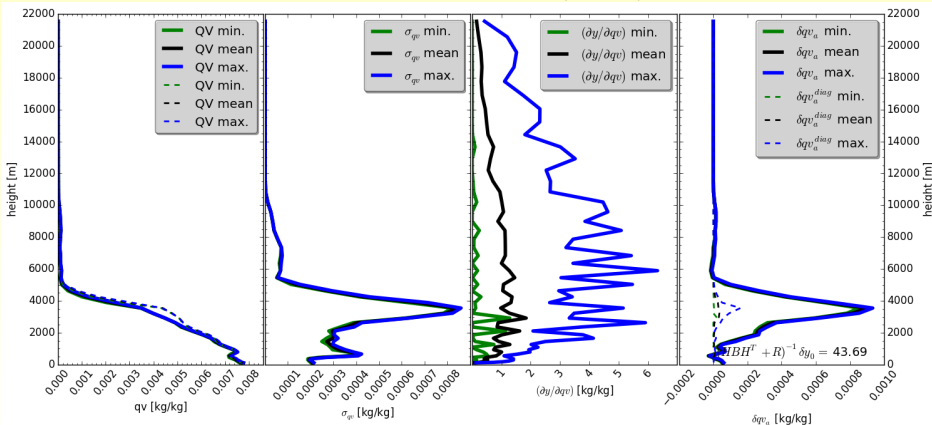
$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \delta \mathbf{y}_0$$

$\mathbf{x}_b, \mathbf{x}_a$

$\mathbf{B}_{\text{diag}}: \text{qv}$

$$\mathbf{H} = \left(\frac{\partial STD}{\partial qv_j} \right)$$

$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T \cdot \mathbf{c}$$



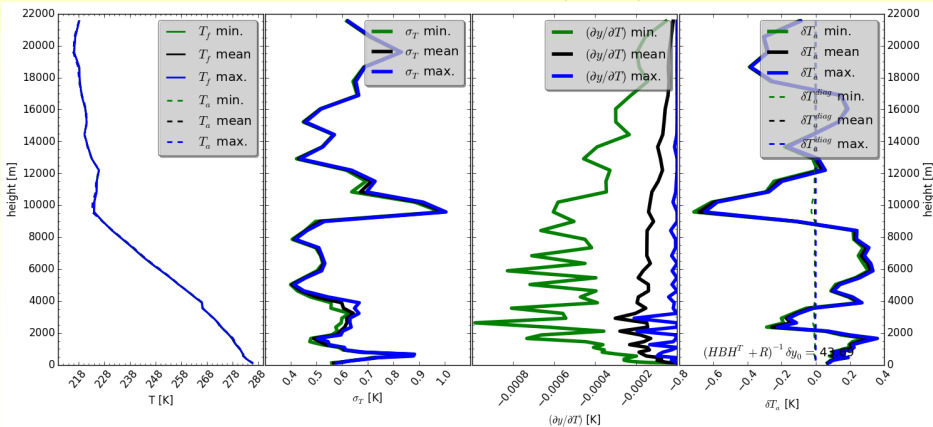
$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \delta \mathbf{y}_0$$

$\mathbf{x}_b, \mathbf{x}_a$

$\mathbf{B}_{\text{diag}}: qv$

$$\mathbf{H} = \left(\frac{\partial STD}{\partial T_j} \right)$$

$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T \cdot c$$



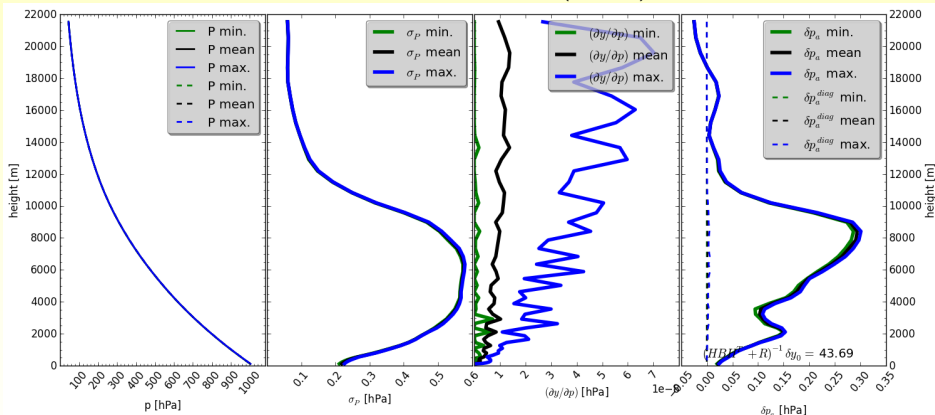
$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \delta \mathbf{y}_0$$

$\mathbf{x}_b, \mathbf{x}_a$

$\mathbf{B}_{\text{diag}}: \text{qv}$

$$\mathbf{H} = \left(\frac{\partial STD}{\partial p_j} \right)$$

$$\delta \mathbf{x}_a = \mathbf{B} \mathbf{H}^T \cdot c$$



- Quality control, black listing
- Correction of some station heights
- Improved bias correction
- Data thinning
- LETKF tuning
 - Localisation of observations
 - Horizontal and vertical localization radii
 - Relative impact of different observation types
- Impact of ZTD, STD and ZTD+STD assimilation
- Combination with latent heat nudging and/or radar
- Observation errors